THE GENERALIZED TLM BASED FINITE-DIFFERENCE TIME-DOMAIN (FD-TD) METHOD AND ITS APPLICATIONS TO FREQUENCY-DEPENDENT AND ANISOTROPIC MEDIA

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Abstract – In this paper, the generalized TLM-based finite-difference time-domain method (FD-TD) is presented for modeling and simulation of frequency-dependent and anisotropic media. The method is the generalized FD-TD form of the TLM symmetrical condensed node and therefore retains the properties of both the FD-TD and TLM methods. It allows the direct use of electromagnetic field quantities in the algorithm and direct treatments of anisotropic media.

I. Introduction

Recently, time-domain solutions for electromagnetic field problems have received growing attention because of their versatility and simplicity. Two widely employed techniques so far are the finite-difference time-domain (FD-TD) method initially formulated by Yee and the transmission-line-matrix (TLM) method proposed by Johns. Although both the FD-TD and TLM methods have been applied widely to solve the similar types of problems, they do possess their own advantages and disadvantages. For instance, the FD-TD method is easily understood and implemented as it is the direct approximation of the Maxwell's equations while TLM requires the transforms between the field quantities and circuit parameters. For most of the cases, the transforms are not complicated but sometimes they are not easily understood. On the other hand, TLM enjoys the use of impulse excitation, Johns matrix, and most significantly, the definition of all the field components at a node. The conventional FD-TD, however, does not intrinsically have these properties. One of the examples is the simulation of an anisotropic medium with non-zero off-diagonal elements. The TLM can be directly applied to this kind of problems [1][2] while the conventional FD-TD can be applied only after introduction of additional averaging schemes in both time and space at every node [3].

In this paper, we propose the generalized TLM based finite-difference time-domain method which is the FD-TD formulation of the three-dimensional TLM

0-7803-3246-6/96/$5.00 © IEEE

symmetrical condensed node. It can basically account for very general electromagnetic problems while retaining the properties of both FD-TD and TLM method. As a result, the formulation allows the direct uses of field quantities and direct simulation of anisotropics.

II. The Generalized TLM Based FD-TD Formulations

In contrast to Yee's FD-TD scheme, in a 3D cell, the field components are now defined as shown in Fig.1. The six field components of $\boldsymbol{E}$, $\boldsymbol{H}$ and their corresponding flux densities, $\boldsymbol{D}$ and $\boldsymbol{B}$, are defined at the center of the 3D cell, while at the grid points on the boundary surface of the 3D cell, only the field components tangential to the surface are considered. As a result, the E-field and H-field components are not separated in space and both the tangential E and H field components are continuous across the interface of two adjacent 3D cells. One of the advantages then is that no special treatments are required for dielectric interfaces as long as the interfaces are placed in between two neighboring nodes.

By finite-differencing Maxwell's equations in respect to the center of a 3D cell, a finite difference formulation for Maxwell's equations can be obtained easily. For example, we can have

$$\frac{D_x^n(i,j,k) - D_x^{n-1}(i,j,k)}{\delta t} + \sigma_{ex}\frac{E_x^n(i,j,k) + E_x^{n-1}(i,j,k)}{2} = -\frac{H_y^{n-\frac{1}{2}}(i,j,k+\frac{1}{2}) - H_y^{n-\frac{1}{2}}(i,j,k-\frac{1}{2})}{\delta z} + \frac{H_z^{n-\frac{1}{2}}(i,j+\frac{1}{2},k) - H_z^{n-\frac{1}{2}}(i,j-\frac{1}{2},k)}{\delta y} \tag{1}$$

The other components can be found in the similar forms. Note here that δx, δz, and δz are not necessarily the same.

For a general media, the constitutive relationships between the flux densities and field intensities can be written as:

$$\mathbf{D} = \mathbf{D}(\mathbf{E}, \mathbf{H}) \tag{2}$$

$$\mathbf{B} = \mathbf{B}(\mathbf{E}, \mathbf{H}) \tag{3}$$

They can represent the anisotropics and frequency-dependence of a medium. By solving the constitutive relationship, $\mathbf{E}$ and $\mathbf{H}$ can then be updated from $\mathbf{D}$ and $\mathbf{B}$.

In order to update the field values at the boundary surface of a 3D cell, a special averaging process in both space and time is performed. For example, one can obtain:

$$\begin{aligned} E_x^{n+\frac{1}{2}}(i,j,k+\tfrac{1}{2}) = \\ 0.5\,[E_x^{n-\frac{1}{2}}(i,j,k-\tfrac{1}{2}) + Z_oH_y^{n-\frac{1}{2}}(i,j,k-\tfrac{1}{2})] + \\ 0.5\,[E_x^{n-\frac{1}{2}}(i,j,k+\tfrac{3}{2}) - Z_oH_y^{n-\frac{1}{2}}(i,j,k+\tfrac{3}{2})] + \\ [E_x^n(i,j,k) + Z_oH_y^n(i,j,k)] + \\ [E_x^n(i,j,k+1) - Z_oH_y^n(i,j,k+1)] \end{aligned} \tag{4}$$

The other components at the boundary surfaces of a 3D cell can be found in a similar way or by simply permutating the indices in the above equations.

III. NUMERICAL RESULTS

The above described method is tested in the two examples. One is to simulate a plane wave incident on a linear dispersive medium with a second-order (Lorentz) dispersion. The second one is to find out the cutoff frequencies of a 3D rectangular waveguide filled with an anisotropic sapphire which has non-zero off-diagonal elements in its permittivity tensor.

For the first case, Figure 2 shows the calculated magnitude and phase of the reflection coefficients as a function of frequency in comparisons with the exact solution [4]. The deviation from the exact solution over the complete range of DC to 3×10^{16} Hz is not visible. For the second case, the anisotropic sapphire filled in the waveguide has the following permittivity tensor

$$[\epsilon] = \epsilon_o \begin{bmatrix} \epsilon_u\cos^2\varphi+\epsilon_v\sin^2\varphi & \frac{1}{2}(\epsilon_u-\epsilon_v)\sin2\varphi & 0 \\ \frac{1}{2}(\epsilon_u-\epsilon_v)\sin2\varphi & \epsilon_v\cos^2\varphi+\epsilon_u\sin^2\varphi & 0 \\ 0 & 0 & \epsilon_z \end{bmatrix}$$

with $\epsilon_u = 9.34$ and $\epsilon_v = 11.49$ [5]. The comparisons between the exact and calculated results are shown in Table I. A good agreement is achieved.

IV. DISCUSSIONS AND CONCLUSIONS

In this paper, the generalized finite-difference time-domain formulation for Maxwell's equations, which is different from that of Yee's scheme, has been proposed. Although the finite-difference scheme presented can be proven to be equivalent to the TLM symmetrical condensed node model, it does not need the transforms to convert the field quantities to the circuit parameters as done in the TLM simulation. In comparison with Yee's FD-TD scheme, the presented method can be applied to simulate anisotropic media directly without introduction of the additional interpolation schemes.

REFERENCES

[1] L. de Menezes and W. J. R. Hoefer, "Modelling anisotropic dispersive materials in TLM", *Proceedings of the First International Workshop on the Transmission Line Matrix (TLM) Modeling Theory and Applications* , pp. 91-94, Victoria, B. C., Aug. 1-3, 1995

[2] J. Huang and K. Wu, "A unified TLM model for wave propagation of electrical and optical structures considering permittivity and permeability tensors", *IEEE Trans. Microwave Theory Tech.* , Vol. 43, No. 10, pp. 2472 – 2477, Oct. 1995

[3] J. Schneider and S. Hudson, "The finite-different time-domain method applied anisotropic material", *IEEE Trans. Antennas*

and Propagat., Vol. 41, No. 7, pp. 994-1000, July 1993

[4] P. Goorjian and A. Taflove, "Direct time integration of Maxwell's equations in linear dispersive media with absorption for scattering and propagation of femtosecond electromagnetic pulses", *Optical Letters,* Vol. 16, No. 18, pp. 1412 – 1414, Sept. 1991

[5] N. G. Alexoupoulos, "Integrated-circuits structures on anisotropic substrates", *IEEE Trans. Microwave Theory Tech.* , Vol. 33, No. 10, pp. 847 – 881, Oct. 1985

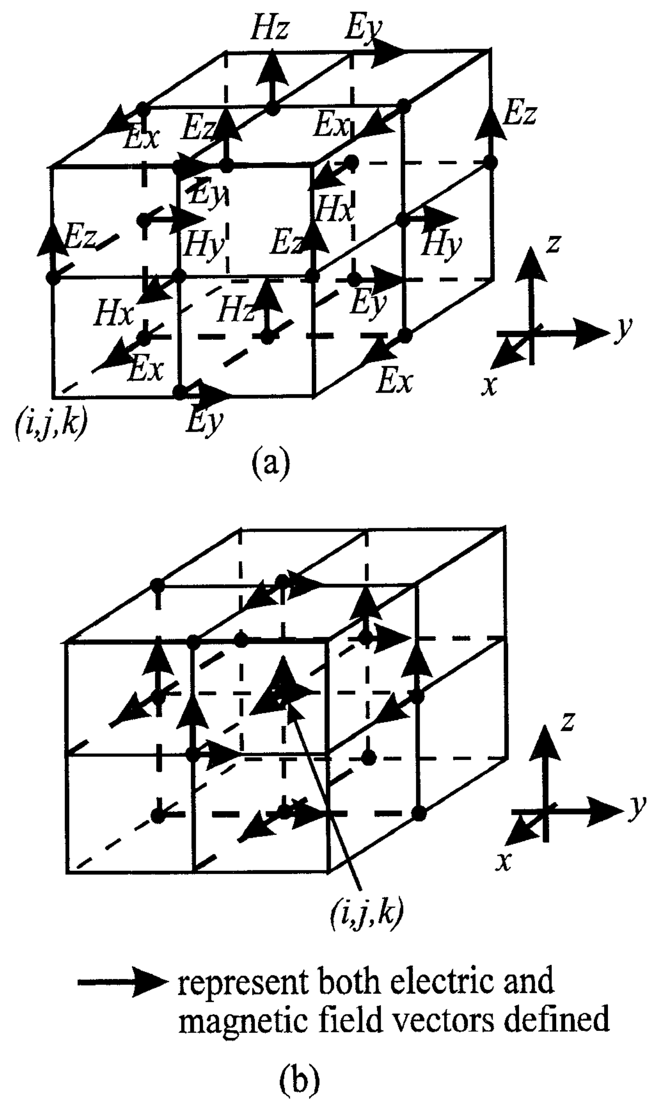

Fig.1 (a) Yee's FD-TD grid arrangement
(b) Proposed FD-TD Grid arrangement

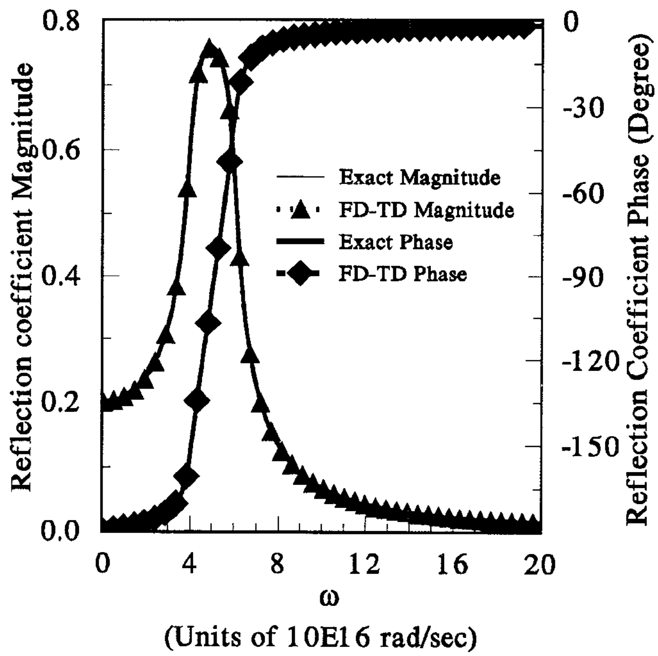

Fig.2 Comparison of the FD-TD and exact results from dc to 3000 THz for the reflection coefficient of a half-plane made of Lorentz medium

Table I
Comparison of the FD-TD and analytical results for the sapphire example

Analytical Solution	Axial Angle (Degree)	The present Method	Error (%)
6.22 GHz	0	6.23 GHz	0.13
6.54 GHz	45	6.55 GHz	0.22
6.90 GHz	90	6.92 GHz	0.04